

$$46) \alpha = (125)(476) \text{ and } \beta = (142)(3567) \text{ of } S_7$$

$$\alpha = (125)(476)$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 7 & 1 & 4 & 6 \end{pmatrix}$$

$$\beta = (142)(3567)$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 5 & 2 & 6 & 7 & 3 \end{pmatrix}$$

$$\alpha\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 7 & 1 & 4 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 5 & 2 & 6 & 7 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 2 & 1 & 5 & 4 & 6 & 3 \end{pmatrix}$$

$$\beta\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 1 & 5 & 2 & 6 & 7 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 7 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 6 & 5 & 3 & 4 & 2 & 7 \end{pmatrix}$$

$$\alpha^2 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 7 & 1 & 4 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 5 & 3 & 7 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 3 & 6 & 2 & 7 & 4 \end{pmatrix}$$

$$\alpha^{-1} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 1 & 3 & 6 & 2 & 7 & 4 \end{pmatrix}$$

4k) Show that  $\mathbb{Q}^*$  under  $\cdot$  is a group for  $r, s \in \mathbb{Q}^*$ , let  $r \cdot s = 3rs$

(i) let  $r, s \in \mathbb{Q}^* \Rightarrow r, s$  are non-zero rationals

Then  $r \cdot s = 3rs$  and  $3rs$  is a non zero rational.  $\therefore r, s \in \mathbb{Q}$

(ii) let  $r, s, t \in \mathbb{Q}^*$

$$r \cdot (s \cdot t) = r \cdot (3st) = 3(r)(3st) = 9rst$$

$$(r \cdot s) \cdot t = (3rs) \cdot t = 3(3rs)t = 9rst$$

$$\therefore r \cdot (s \cdot t) = (r \cdot s) \cdot t \text{ for all } r, s, t \in \mathbb{Q}^*$$

(iii) let  $r \in \mathbb{Q}^*$

let's consider  $t \in \mathbb{Q}^*$  such that  $r \cdot t = r$

$$r \cdot t = r$$

$$\Rightarrow 3rt = r$$

$$\text{or } t = \frac{1}{3}$$

$$\therefore r \cdot \frac{1}{3} = 3(r)\left(\frac{1}{3}\right) = r$$

$$\frac{1}{3} \cdot r = (3)\left(\frac{1}{3}\right)(r) = r$$

$$\therefore r \cdot \frac{1}{3} = \frac{1}{3} \cdot r = r \text{ for all } r \in \mathbb{Q}^*$$

$\frac{1}{3}$  is the identity element

(iv) let  $r \in \mathbb{Q}^*$  and there exists an element  $s \in \mathbb{Q}^*$  such that  $r \cdot s = \frac{1}{3}$

$$r \cdot s = \frac{1}{3}$$

$$\Rightarrow 3rs = \frac{1}{3}$$

$$\text{or } s = \frac{1}{9r}$$

$$\text{Now } s \cdot r = s\left(\frac{1}{9r}\right)(r) = \frac{1}{3}$$

$$\therefore r \cdot \frac{1}{9r} = \frac{1}{9r} \cdot r = \frac{1}{3} \text{ for all } r \in \mathbb{Q}^*$$

Therefore,  $\frac{1}{9r}$  is the inverse of  $r$  and

$r$  is arbitrary element of  $\mathbb{Q}^*$

$\therefore$  from (i), (ii), (iii), (iv)  $\mathbb{Q}^*$  is a group under  $\cdot$